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Magneto-viscous effect on thermal convection thresholds in an Oldroyd magnetic fluid

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ABSTRACT

In magnetic fluids the viscosity can depend on an external magnetic field. We theoretically investigate the influence of this magneto-viscous effect on the thermal convection thresholds for viscoelastic ferrofluids, which are described by a linear Oldroyd model. Such a system is influenced by a static magnetic field not only via the Kelvin force, but also through the magneto-viscous effect. In particular, we find that these two contributions compete oppositely when the threshold for the oscillatory instability is considered. While the Kelvin force tends to decrease the critical Rayleigh number, the magneto-viscous effect increases it. The critical properties at the onset of the oscillatory instability are discussed as a function of the viscoelastic parameters, the external field strength, and the magneto-viscous coefficient. The transition between the stationary and the oscillatory instability is only slightly affected by the magneto-viscous effect. Examples for codimension-2 lines are given.

1. Introduction

The purpose of the present article is to analyze the influence of a magnetic field dependence of the viscosity on the convective threshold of viscoelastic magnetic fluids. As model systems we consider ferrofluids [1], suspensions of ferromagnetic particles in a carrier liquid, in the coarse-grained approximation, where particle diffusion and thermodiffusion are neglected. The magnetic fluid properties can then be modeled as electrically nonconducting superparamagnets [2]. Such suspensions are often slightly viscoelastic, which we describe by a linear Oldroyd model that contains relaxation of the stress and retardation of the "strain rate". The use of such models is rather popular, since viscoelasticity occurs in the form of a pseudo-constitutive equation and the structure of the hydrodynamic equations remains unchanged. The more physical description, suitable to generalizations to more complicated systems or to the nonlinear domain, introduces the elastic free energy in terms of the strain tensor, which constitutes an additional hydrodynamic degree of freedom. The relaxation of the latter describes viscoelasticity as transient elasticity. In the linear domain, however, both types of descriptions are basically equivalent.

For magnetic suspensions, in particular highly concentrated ones, the visco-elastic properties can be magnetic-field dependent. In this study, however, we concentrate on the magneto-viscous effect, *i.e.* the magnetic field dependence of the viscosity [3]. The maximum increase of the viscosity that can be obtained is about 30%. Although this looks like a small effect, its influence on the bifurcation behavior can be large, since this non-Boussinesq contribution breaks some symmetries of the underlying hydrodynamic equations. In addition, it constitutes a magnetic field influence directly on the (dissipative) dynamics, in contrast to the more familiar magnetic Kelvin force that acts on the static properties. The magneto-viscous effect has been experimentally measured in different types of magnetic fluids [4–8]. For instance, the technique of small angle scattering has been employed in the case of magnetite-base ferrofluids leading to a microscopic explanation [8]. In addition, this effect can have technological applications in pipe systems, dynamic sealing, as well as in servo-rheological devices [9]. Furthermore, theoretical descriptions of the field-dependence of the viscosity have been given using a polydisperse statistical mechanical approach [10,11] as well as a continuous hydrodynamic one [12].

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We will discuss the linear stability of stationary and oscillatory instabilities driven by an applied temperature gradient and an applied magnetic field. We deal with the case of realistic boundary conditions, and we numerically solve the dynamic equations using a collocation spectral method in order to determine the eigenfunctions and eigenvalues and consequently the convective thresholds. The paper is organized as follows: In Section 2, the basic hydrodynamic equations for viscoelastic magnetic fluid convection are presented. In Section 3 the linear stability analysis of the conduction state is performed. Finally, conclusions are presented in Section 4.

2. Basic equations

We consider a layer of thickness d of viscoelastic magnetic fluid in a vertical (z direction) temperature gradient, $\beta = \Delta T/d$, antiparallel to the gravitational field (strength g). An external magnetic field $\boldsymbol{H}_0$ is assumed along the vertical direction. Within the Boussinesq approximation the fluid is incompressible ($\mathrm{div}\,\boldsymbol{v} = 0$) and the density $\rho = \rho_0$ is constant, except for the buoyancy force, where a linear dependence on the temperature T is assumed

$$\rho = \rho_0(1 + \alpha_T\,\delta T) \quad (1)$$

neglecting magnetic buoyancy, where α_T is thermal expansion coefficient. Here, we consider that these constants take the values $\rho_0 = 10^3$ kg/m^3 and $\alpha_T = 10^{-3}$ 1/K [1]. The magnetization is treated statically and does not have its own dynamics. In particular, in the magnetization $\boldsymbol{M}$ thermal and nonlinear magnetic field effects are taken into account [16]

$$\delta\boldsymbol{M} = \chi_0\,\delta\boldsymbol{H} + \boldsymbol{H}_0(\chi_T\,\delta T + \chi_H \boldsymbol{H}_0 \cdot \delta\boldsymbol{H}), \quad (2)$$

where $\{\chi_0, \chi_T, \chi_H\}$ are the respective susceptibilities for which we take the values $\{1.9, 5\times10^{-2}$ 1/K, 10^{-8} m^2/A$^2\}$. Here, we describe viscoelasticity by the well-known linear Oldroyd model

$$(1 + \lambda_1\partial_t)\tau_{ij} = 2\nu_{eff}(1 + \lambda_2\partial_t)A_{ij}, \quad (3)$$

relating the stress τ_{ij} and its relaxation (λ_1) with the strain rate $A_{ij} = (1/2)(\nabla_j v_i + \nabla_i v_j)$ and its retardation (λ_2). The effective viscosity $\nu_{eff} = \nu_1(\lambda_1/\lambda_2)$ is different from the Newtonian viscosity ν_1. Describing linear viscoelasticity by a relaxing strain field [17–19] and a finite elastic (plateau) modulus K_1, one finds $K_1 = 2\nu_1(1/\lambda_2 - 1/\lambda_1)$. Since $K_1 > 0$ and $\nu_1 > 0$ for thermodynamic reasons, this implies the restriction $\lambda_1 > \lambda_2$ on the Oldroyd model and shows that the limit $\lambda_1 \neq 0$ and $\lambda_2 \to 0$ (often referred to as the Maxwell model) is ill-defined, since it would imply a diverging elastic modulus. Using nonlinear viscoelasticity (for an application to thermal convection cf. [20] and to general flow situations cf. [21,22]) one can show [23] that many (nonlinear) phenomenological models have similar problems and restrictions when thermodynamics is applied.

The magneto-viscous effect [3] shows up in the viscosity ν_{eff}, which is not constant, since it depends on the magnetic field

$$\nu_{eff} = \nu_0(1 + \eta\boldsymbol{H}^2) \quad (4)$$

with ν_0 the (effective) viscosity at zero field, and with a positive magneto-viscous coefficient η. The range for η will be $(10^{-11}–10^{-7})$ m^2/A^2. At least for small fields this form is required by symmetry. Indeed, measurements on dilute ferrofluids confirm this field dependence for up to field strengths of $H \approx 10$ kA/m. Generally, the viscosity is a nonlinear function of the magnetic field amplitude [3,6–8], but the linear approximation is used frequently in theoretical works [13–15]. For very high fields the influence of the magnetic field saturates [3]. In the case of magneto-rheological fluids not only the effective viscosity, but also visco-elasticity is dramatically increased by an external field, and qualitatively new effects, like yield stress and thixotropy arise. Those effects will not be considered here. The remaining part of the hydrodynamics is that of an incompressible superparamagnet and has been given and applied to thermal convection previously [16,24–27]. It is well known that such system has a quiescent, flow free ground state that is purely heat conducting

$$\boldsymbol{v}_{con} = 0 \quad (5)$$

$$T_{con} = T_0 - \beta z \quad (6)$$

$$H_z^{con} = H_0(1 + \xi\beta z) \quad (7)$$

with $\xi = \chi_T/(1 + \chi_0 + \chi_H H_0^2)$. The inhomogeneity in Eq. (7) is due to the magnetic properties of the fluid and follows from the general magnetostatic Maxwell continuity conditions across the horizontal boundaries, $\boldsymbol{H}_\perp = \boldsymbol{H}_\perp^{ext}$ and $B_z = B_z^{ext}$ for the internal and external fields.

For the deviations from this ground state, $v_i, \theta = T - T_{con}$, and the magnetic potential ϕ with $\boldsymbol{H} = \boldsymbol{H}_0 - \nabla\phi$, linear dynamic equations have been previously derived with the result (in dimensionless form) [24–26]

$$\nabla_i v_i = 0 \quad (8)$$

$$\frac{1}{P}\partial_t v_i = -\nabla_i p_{eff} + \nabla_j\tau_{ij} + \delta_{iz}Ra\,[\theta + M_1(\theta - \partial_z\phi)] \quad (9)$$

$$\partial_t\theta = w + \nabla^2\theta \quad (10)$$

$$\partial_z\theta = (\partial_{zz}^2 + M_3\nabla_\perp^2)\phi \quad (11)$$

$$\nabla^2\phi_{ext} = 0 \quad (12)$$

where $\nabla_\perp^2 = \partial_{xx}^2 + \partial_{yy}^2$.

We have used the characteristic scales, d for length, d^2/κ for time (with κ the thermal diffusivity), βd for temperature, κ/d for velocity, and $\beta d\xi H_0$ for the magnetic fields. The Rayleigh number $Ra = \alpha_T g\Delta T d^3/\kappa\nu_0$ that contains the thermal driving and acts as the primary control parameter. The Prandtl number $P = \nu_0/\kappa$ relates the viscous with the thermal diffusion properties of the fluid. There are two standard magnetic numbers, $M_1 = \mu_0\beta\chi_T^2 H_0^2/(\rho_0 g\alpha_T[1 + \chi_0])$ characterizing the magnetic force relative to buoyancy and $M_3 = (1 + \chi_0)/(1 + \chi_0 + \chi_H H_0^2)$ the nonlinearity of the magnetization. M_1 is a secondary control parameter, since the conducting state can also be driven into instability by a large enough M_1.

The viscoelastic Eq. (3) reads in linearized form and assuming $\xi\beta d \ll 1$

$$(1 + \Gamma\partial_t)\tau_{ij} = 2(1 + a_2 + a_1 z)(1 + \Lambda\Gamma\partial_t)A_{ij}, \quad (13)$$

and contains two new dimensionless numbers related to the magneto-viscous effect, $a_2 = \eta H_0^2$ and $a_1 = 2\eta\,\xi\beta d H_0^2$. The former directly characterizes the strength of the magneto-viscous effect, while the latter describes its influence due to the inhomogeneity of the conductive ground state. The ratio $a_1/a_2 = 2\xi\beta d$ might be small, but the different spatial symmetry requires keeping both terms in Eq. (13). Generally, there is also a quadratic contribution in Eq. (13), $\sim a_3 z^2$, which is of the same spatial symmetry as the constant terms. Averaging $\langle z^2\rangle = (1/2)d^2$ shows that it can be neglected compared to $1 + a_2$.

Viscoelasticity is characterized by the Deborah number $\Gamma = \lambda_1\bar{\kappa}/d^2$ and the relaxation ratio $\Lambda = \lambda_2/\lambda_1$. Since $\lambda_{1,2}$ are positive, so are Γ and Λ. The Newtonian case is recovered by $\Gamma \to 0$ and $\Lambda \to 1$ (no elasticity), while the limit $\Lambda \to 0$ is unphysical.

For the numerical solutions we have to specify the dimensionless numbers. The Ra can vary over several orders of magnitude, while a typical value for P in viscoelastic fluids is $P \sim 10^0 - 10^3$. For the magnetic numbers we consider the range $M_1 \sim 10^{-4} - 10$ and $M_3 \sim 1$ [24,28]. For aqueous suspensions it is suggested that the Deborah number is about $\Gamma \sim 10^{-3} - 10^{-1}$ [29–32], but for other kinds of viscoelastic fluids the Deborah number can be as large as $\Gamma \sim 10^3$. Unfortunately, no experimental data are available for the relaxation ratio, so we treat Λ as arbitrary in the range $(0, 1)$.

3. Linear stability analysis

3.1. Mathematical procedure

The flow Eqs. (8), (9) and (13) can be combined to a single equation for w, the z component of the velocity, by a standard procedure, applying appropriately the *curl* and *div* operators as well as $(1 + \Gamma\partial_t)$ with the result

$$(1 + \Gamma\partial_t)\left(\frac{1}{P}\nabla^2 w - Ra\nabla_\perp^2(\theta + M_1[\theta - \partial_z\phi])\right)$$
$$= (1 + \Lambda\Gamma\partial_t)\nabla^2\Big((2a_1\nabla_z + [1 + a_2 + a_1 z]\nabla^2)w\Big) \tag{14}$$

The remaining variables relevant for the linear analysis can be written as a vector field $\boldsymbol{u} = (\theta, \phi, w)^T$. Using standard techniques [33], the spatial and temporal dependencies of $\boldsymbol{u}$ are separated using a normal mode expansion

$$u(r, t) = U(z)\exp[ik \cdot r_\perp + st], \tag{15}$$

with $U(z) = (\Theta(z), \Phi(z), W(z))^T$ and k being the horizontal wave vector of the perturbations, $r_\perp$ the horizontal position vector, and $s = \sigma + i\Omega$ the complex eigenvalue. The latter contains the linear growth rate, σ, and the frequency, Ω, of the perturbation. With this ansatz Eqs. (10), (11), (14) are reduced to a set of coupled ordinary differential equations

$$D^2\Theta = (k^2 + s)\Theta - W \tag{16}$$
$$D^2\Phi = M_3 k^2\Phi + D\Theta \tag{17}$$

$$\mathscr{L}_{\mathscr{F}}(D^2 - k^2)^2 W - \left(\frac{sQ}{P} - 4\upsilon_1 D\right)(D^2 - k^2)W \tag{18}$$
$$= k^2 QRa([M_1 + 1]\Theta - M_1 D\Phi) \tag{19}$$

where D denotes the spatial differentiation d/dz of the functions U. The abbreviation $Q = (1 + s\Gamma)/(1 + s\Lambda\Gamma)$ contains the influence of viscoelasticity and is equal to one in the Newtonian case. The magnetic field dependence of the viscosity shows up in Eq. (19) in the contribution $\mathscr{L}_{\mathscr{F}} \equiv 1 + a_2 + a_1 z$ with the non-autonomous term $\sim a_1$ and the constant $\sim a_2$. The evaluation of their influence is the main target of this manuscript.

The differential equations have to be supplemented by boundary conditions that are for viscous as well as viscoelastic fluids

$$W = DW = \Theta = 0, \tag{20}$$

at the two horizontal rigid boundaries. These BCs produce a more realistic result than the common free-free BCs. In addition, in the case of a finite magnetic permeability χ_b of the rigid boundaries, the scalar magnetic potential must satisfy

$$(1 + \chi_b)D\Phi \pm k\Phi = 0, \tag{21}$$

at $z = \pm d/2$, respectively, as shown by Finlayson [16] matching the solution of the bulk magnetic potential with the external magnetic potential. Note that only in the limit when χ_b tends to infinity, Eq. (21) simplifies to $D\Phi = 0$. For large values of the external magnetic field (of the tens of thousands A/m) χ_b is vanishing, while for moderate values of the external magnetic field the parameter χ_b takes values close to unity, depending on the material. The numerical results below are obtained for $\chi_b = 1$.

In order to solve Eqs. 16,17,19 with these realistic boundary conditions, we use a spectral collocation method. Spectral methods ensure an exponential convergence to the solution and are the best available numerical techniques for solving simple eigenvalue – eigenfunction problems. Here, we follow the technique of collocation points on a Chebyschev grid as described in [34]. The collocation points (Gauss-Lobato) are located at height $z_j = \cos(j\pi/N)$ where the index j runs from $j = 0$ to $j = N$. Note that here the z-variable ranges from -1 to $+1$ and one has therefore to rescale Eqs. 16,17,19 accordingly, because the physical domain is defined in the range $(-1/2, +1/2)$. We use $N = 14$ collocation points in the vertical direction, for which the equations and the boundary conditions are expressed. We have checked that using $N = 20$ collocation points only modifies the fourth or fifth significant digit of the result. By using the collocation method, the set of differential Eqs. 16,17,19 is transformed into a set of linear algebraic equations. The eigenfunctions $(\Theta(z), \Phi(z), W(z))$ are transformed into eigenvectors defined at the collocation points. The Rayleigh number Ra is again used as the eigenvalue of the problem. After this stage of discretization, one is left with a classical generalized eigenvalue–eigenvector problem that can easily be solved using the Matlab routine "*eig*" [35].

It is well-known that Newtonian magnetic fluids of the kind considered here only show a stationary instability. This instability is not at all influenced by visco-elasticity. However, visco-elasticity allows for an oscillatory instability, on which we concentrate in the following. Since the eigenvalue problem is complex, one has to make sure that Ra (as being a physical quantity) is a real number by choosing a correct value for Ω. Therefore, one is left with a triplet $\{Ra, k, \Omega\}$ that defines the marginal stability condition (for a given value of the horizontal wavenumber k). This procedure is repeated for several values of k leading to the marginal stability curve Ra versus k. The minimum of this curve defines the critical Ra_{oc} and k_c, and the corresponding value for the critical frequency Ω_c.

3.2. Discussion of results

In Fig. 1 we discuss the influence of a magnetic field on the critical properties of the oscillatory instability. The latter is obtained by choosing the viscoelastic parameters ($\Lambda = 0.5,\ \Gamma = 0.1$), appropriately. For low fields there is no influence. For higher fields the threshold Ra_{oc} decreases with increasing field strength, if no magneto-viscous effect is present ($\eta = 0$). In that case a magnetic field only acts on the system via the static Kelvin force. It is known that in this case the quiescent ground state is destabilized and Ra_{oc} is reduced. Obviously, the magneto-viscous effect has an opposite influence, since increasing the viscosity stabilizes the quiescent ground state. For intermediate values of η the competition of these two effects leads to a non-monotonous behavior of Ra_{oc}, while for larger η the threshold increases (there is a saturation not shown here). For the critical frequency Ω_c the influence of the magneto-viscous effect is less pronounced, since even without the latter Ω_c increases with the field strength. There is only a very small influence of η on the critical wavenumber k_c.

A rather similar scenario is found, when the critical properties of the oscillatory instability are discussed as a function of the magneto-viscous effect, as it is shown in Fig. 2. Choosing a value for the external field ($H_0 = 5$ kA/m) large enough to be of influence, but still small enough for Eq. (4) to be applicable, Ra_{oc} (Ω_c) sharply (rather slightly) increases for large η, while k_c is almost unaffected by η. This is shown for viscoelastic parameters typical for the oscillatory instability ($\Gamma = 0.1,\ \Lambda = 0.25$ and 0.5).

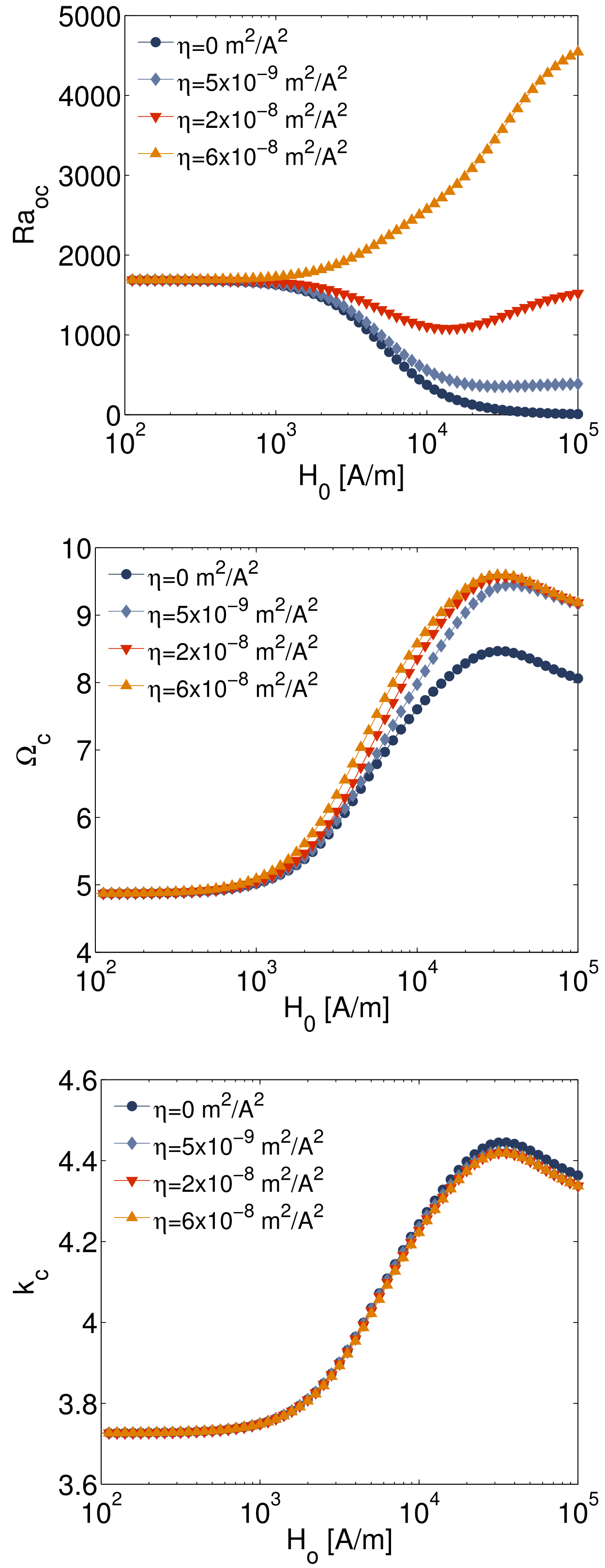

Fig. 1. The critical properties of the oscillatory instability, Ra_{oc}, k_c, and Ω_c, as a function of an external magnetic field, H_0, for different values of the magneto-viscous coefficient η. The fixed parameters are: $\Gamma = 0.1, \Lambda = 0.5, P = 10, d = 1$ mm, $\mu_0 = 4\pi \times 10^{-7}\mathrm{N/A^2}, \rho = 1000\ \mathrm{kg/m^3}, g_0 = 9.8\ \mathrm{m/s^2}, \alpha_T = 10^{-3}\ 1/\mathrm{K}, \chi_0 = 1.9,\ \chi_b = 1,\ \chi_H = 10^{-8}\ \mathrm{m^2/A^2}$, and $\chi_T = 5 \times 10^{-2}\ 1/\mathrm{K}$.

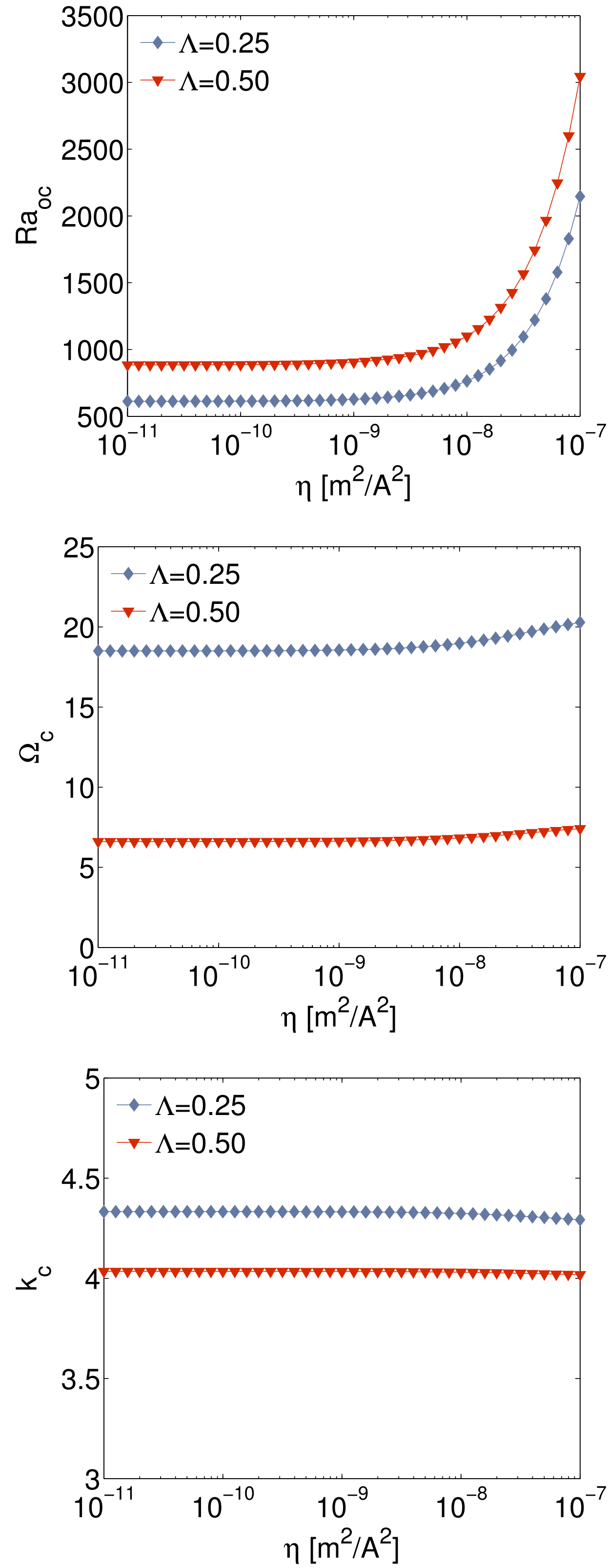

Fig. 2. The critical properties of the oscillatory instability as a function of the magneto-viscous coefficient η at $H_0 = 5 \times 10^3$ A/m for two values of the relaxation ratio $\Lambda = 0.25$ (indicated by diamond symbols) and $\Lambda = 0.5$ (indicated by triangles). The other fixed parameters are the same of Fig. 1.

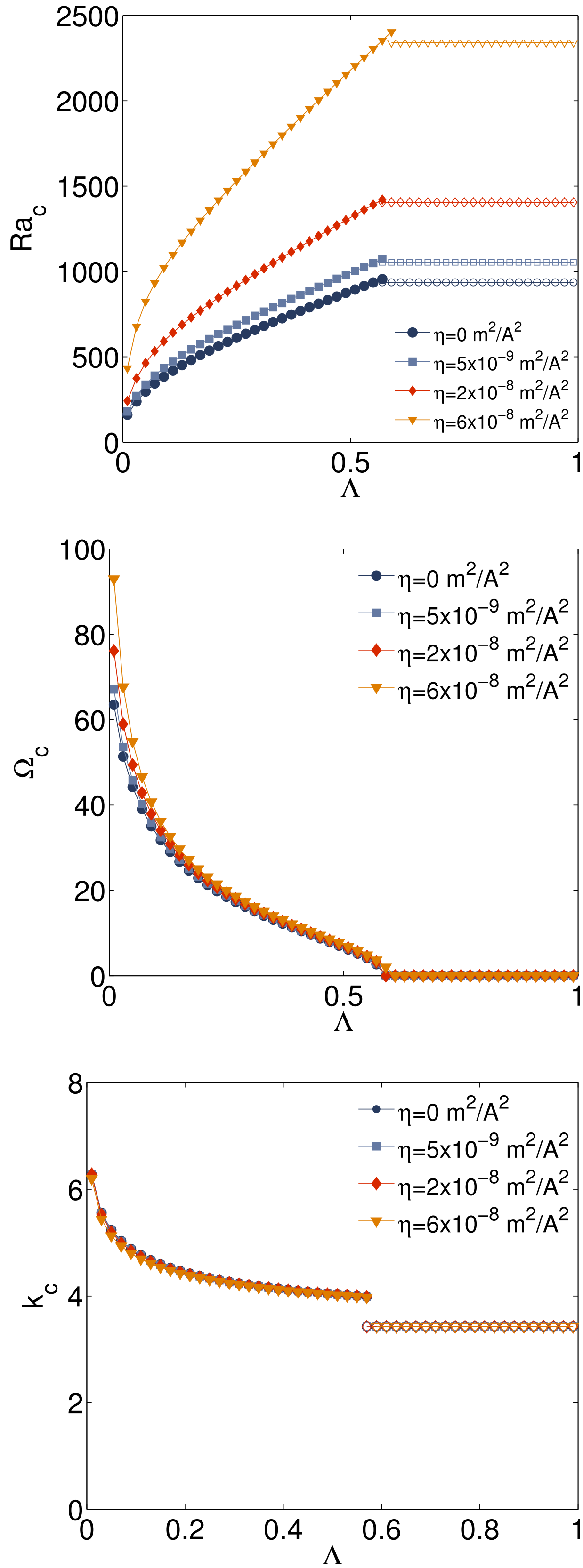

Fig. 3. The critical properties Ra_c, k_c, and Ω_c as a function of the relaxation ratio Λ for different values of the magneto-viscous coefficient η at $H_0 = 5 \times 10^3$ A/m. The other fixed parameters are the same of Fig. 1. The hollow (filled) symbols correspond to the stationary (oscillatory) case.

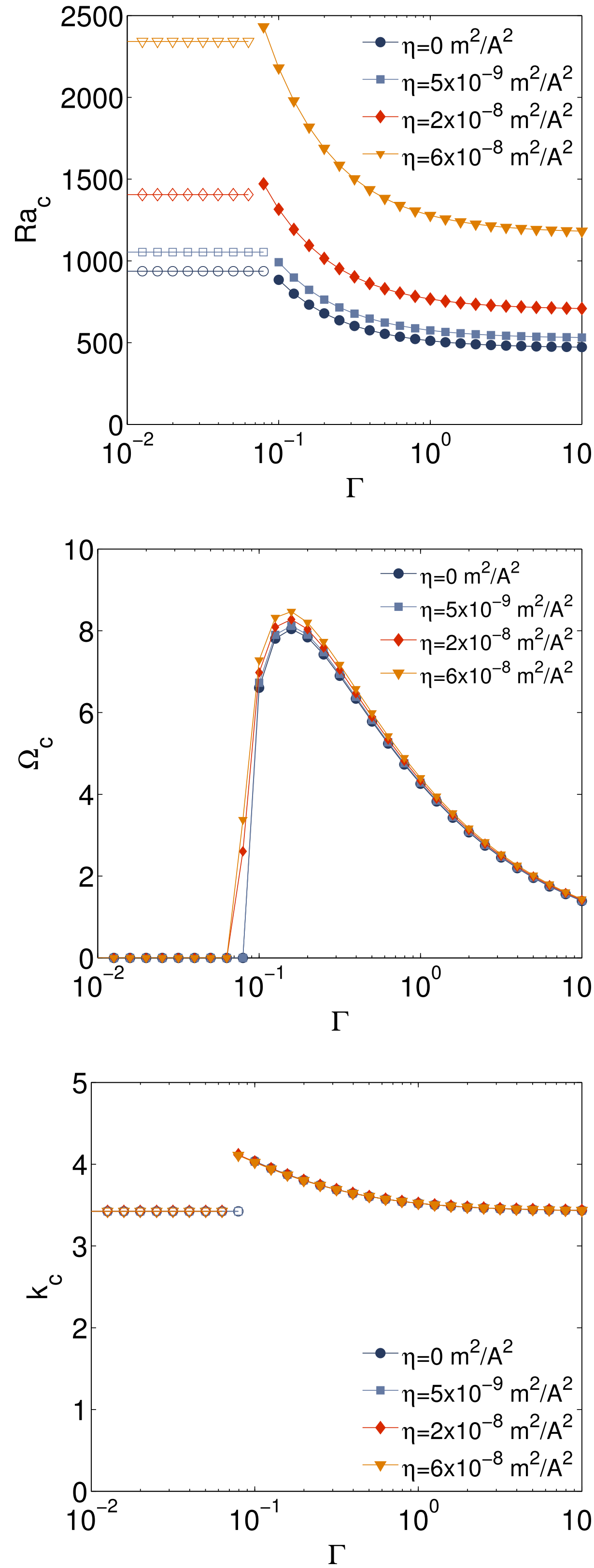

Fig. 4. The critical properties Ra_c, k_c, and Ω_c as a function of the Deborah number Γ for different values of the magneto-viscous coefficient η at $H_0 = 5 \times 10^3$ A/m. The other fixed parameters are the same of Fig. 1. The hollow (filled) symbols correspond to the stationary (oscillatory) case.

In Figs. 3 and 4 we show the influence of η on the transition from the stationary to the oscillatory instability that is obtained by varying the viscoelastic parameters Λ and Γ. In the former case lower (larger) values of Λ (with $\Gamma = 0.1$ fixed) lead to the oscillatory (stationary) instability. Although the critical Rayleigh numbers increase for both instability types with increasing magneto-viscous effect, the value of Λ, where the cross-over takes place, is unaffected by η. Again, there is only a slight effect on Ω_c and almost no effect on k_c due to η. The same scenario is found in Fig. 4, where Γ is varied (and $\Lambda = 0.5$ fixed) with the difference that the stationary (oscillatory) instability is obtained for smaller (larger) values of Γ. In both cases $H_0 = 5$ kA/m.

For specific values of the parameters, the critical Rayleigh values of the two instabilities, Ra_{sc} and Ra_{oc}, respectively, can coincide leading to a codimension-2 point. An example is shown in Fig. 5, where we have fine-tuned the value of $\Lambda = 0.56515$ ($\Gamma = 0.1$) in order to get coincidence of the critical Rayleigh numbers. In this particular case we get $Ra_{oc} = Ra_{sc} = 2341.78$, $k_{oc} = 3.977, k_{sc} = 3.424$ and $\Omega_c = 3.9798$. Once we have obtained a codimension-2 point, we can follow this point in parameter space to get a line of codimension-2 points. Two codimension-2 lines are displayed in Fig. 6 corresponding to two different values of the external magnetic field, $H_0 = 10^3$ A/m and 5×10^3 A/m, respectively. These lines in the (Λ, Γ) plane indicate the location of codimension-2 points. In addition, we shown in Fig. 7 the corresponding critical properties associated with these codimension-2 points. It is interesting to

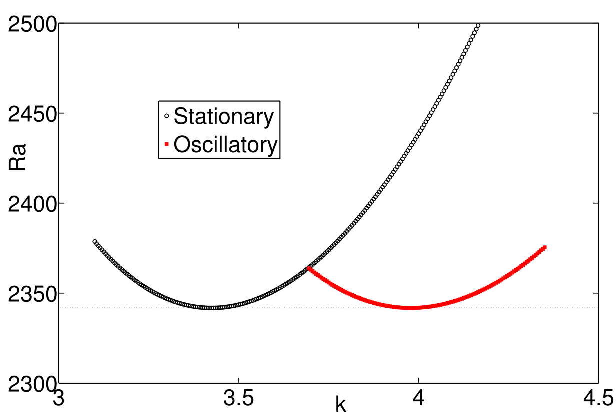

Fig. 5. Example of a codimension-2 point obtained for the special parameter values $\eta = 6 \times 10^{-8}\,\mathrm{m}^2/\mathrm{A}^2$, $H_0 = 5 \times 10^3$A/m, $\Gamma = 0.1$, and $\Lambda = 0.56515$. The other fixed parameters are the same of Fig. 1.

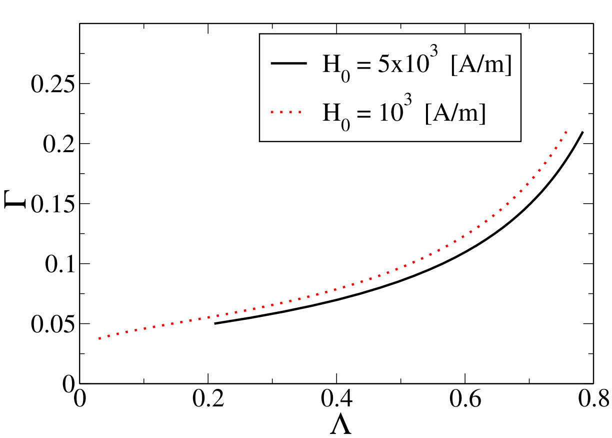

Fig. 6. Representation of the codimension-2 curve in the (Λ, Γ) plane for two different values of the magnetic field $H_0 = 10^3$A/m and $H_0 = 5 \times 10^3$ A/m at $\eta = 6 \times 10^{-8}\,\mathrm{m}^2/A^2$. The other fixed parameters are the same of Fig. 1.

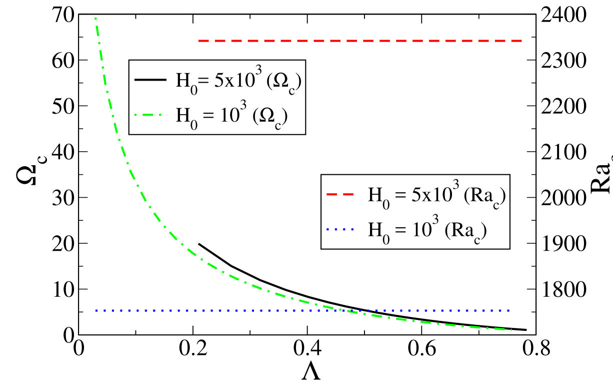

Fig. 7. Critical properties, Ω_c on the left ordinate and $Ra_{oc} = Ra_{sc} = Ra_c$ on the right ordinate, associated with the codimension-2 curves of Fig. 6 are shown as a function of Λ. The corresponding values of Γ follow from Fig. 6.

note that in Fig. 7 the critical Rayleigh values only depend on the external magnetic field, but not on Λ, as we move along the codimension-2 curve.

4. Final remarks

In soft matter science the direct measurement of the various material parameters of complex fluids is sometimes rather involved. Alternatively, one can look how a special material parameter influences the instability scenario, when the material is driven out of equilibrium. As an example we have shown here that the magneto-viscous parameter η describing the magnetic field dependence of the shear viscosity has a prominent influence on the onset of oscillatory instability in visco-elastic ferrofluids. For a large enough values of η the threshold increases with the external magnetic field, while for η small or zero the external field decreases the onset. In the latter case the static Kelvin force exclusively carries the field influence, while a finite η leads to an additional field influence in the dissipative dynamics. For intermediate η values a non-monotonous field dependence is found for the threshold. These results are obtained within linear stability analysis.

In the context of nonlinear dynamics the magneto-viscous effect is non-Boussinesq giving rise to a non-autonomous stability problem and to the breaking of spatial up-down symmetry. As a consequence, one can expect non-trivial pattern selection and switching between different patterns (*e.g.* roll and hexagonal convection) by varying the external magnetic field. This requires, however, a nonlinear stability analysis including a nonlinear modeling of the visco-elasticity [20].

Another extension of the present work could take into account the field dependence of the visco-elastic parameters, like the elastic plateau modulus K_1 and the strain relaxation λ_1. The former is expected to increase with the field, since internal transient elastic structures are enhanced by the field. While the shear viscosity η increases under a (static) magnetic field, since (in the simplest model) the field hinders the mutual rotation of the magnetic particles, *a priori* predictions for the field dependence of λ_1 are not possible. Such a scenario is important for magneto-rheological systems, where, however, additional qualitatively new effects, like yield stress and thixotropy, are to be considered.

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